

Multiplication and Division Facts

Greatest Common Factor and

Least Common Multiple

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January 31, 2014

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Multiplying & Dividing Two Numbers in decimal notation, the Easy Way

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Multiplication of two numbers is a struggle for middle school students from class grade five thru eleven. Often student's parents are unable to perform this multiplication correctly. Here we have described a simple method, a step by step process to multiply two decimal numbers ensuring that result is error free.

An important characteristic of numbers in any system of radix (base) is it can be expressed as sum of numbers those are multipliers of base. For instance, number 45 in 10 based number system is expressed as $4 \times 10^1 + 5 \times 10^0$. Also any digit that is multiplied by some power of 10 is written as the digit with the same number of zeroes as the power number (exponent). In above example, $4 \times 10^1 = 4 \times 10 = 40$ and $5 \times 10^0 = 5 \times 1 = 5$. We will apply this property to develop a step by step process to multiply two numbers that have fractional part expressed as a decimal fraction. First we will state the steps required to multiply two numbers and then we will illustrate the process by describing an example situation. Further we will explicate, how the procedure can be extended for division of two numbers those have decimal fractions. Our technique is based on so called Booth's algorithm that is frequently used to speed up multiplication in Arithmetic Logic Unit of an advanced computer system.

Steps to multiply M1 & M2 to find result, product.

- Step S(1): Write multiplication expression in-line (this will be clear when you look at the example) after expressing second number in base 10 notation. Ignore decimal points in both M1 and M2.
- Step S(2): Count number of decimal places on right side after decimal point (DP) in first number, p1.
If no decimal point then p1 is zero. The place count is not zero only if any digit on rhs of DP is nonzero.
- Step S(3): Count the number of decimal places on right side of decimal point in second number, p2. Add the two places counted. Place decimal point in result as per places computed in S(2) & S(3).
- Step S(4): Create partial sums by multiplying each digit of second number to each digit of the first Number starting from right most digit of the second number proceeding the right to the left digits in succession. At this time ignore decimal points in both numbers. As you advance digits of second number during multiplication, place an x in the partial sum. Treat x as if it is a zero (This step will be clear in illustration) while performing addition.
- Step S(5): Add all partial sums and place the decimal point in the result.

S(5)⇒ Place decimal point, position $p1 + p2$



We will describe the procedure using a numerical example. Let us multiply $25.52 \times 22.045 = 562.58840$

S(1) $2552 \times (2 \times 10000 + 2 \times 1000 + 0 \times 100 + 4 \times 10 + 5 \times 1)$, S(2) count 2, S(3) count 3, total 5

$$\text{S(4)} \quad 2552 \times (5 \times 1) = \quad 1 \ 2 \ 7 \ 6 \ 0$$

$$2552 \times (4 \times 10) = \quad 1 \ 0 \ 2 \ 0 \ 8 \ x$$

$$2552 \times (0 \times 100) = \quad 0 \ 0 \ 0 \ 0 \ x \ x$$

$$2552 \times (2 \times 1000) = \quad 5 \ 1 \ 0 \ 4 \ x \ x \ x$$

$$2552 \times (2 \times 10000) = \quad \underline{5 \ 1 \ 0 \ 4 \ x \ x \ x \ x}$$

Here x's are position (place) markers.

Above x is multiplication $5 \ 6 \ 2 \ 5 \ 8 \ 8 \ 4 \ 0$

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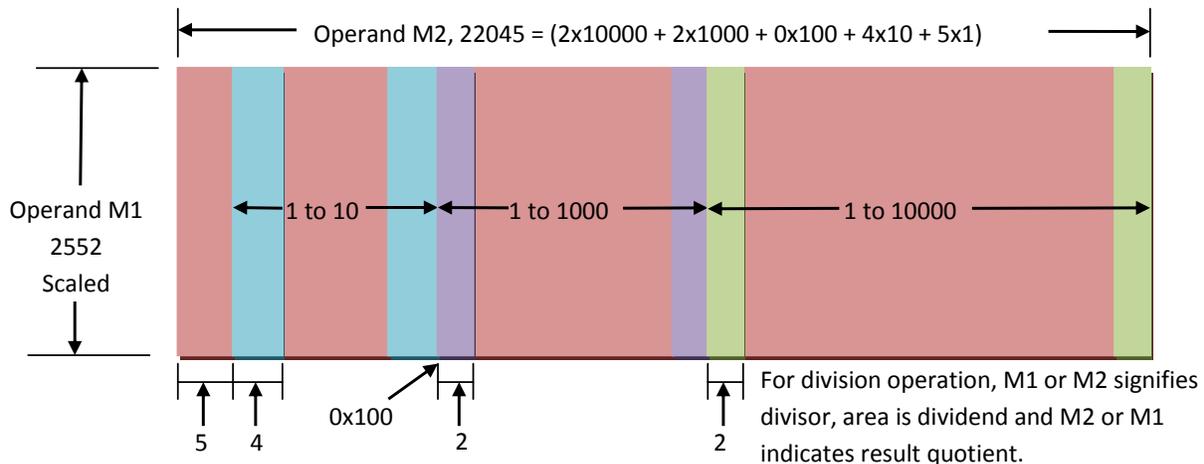


Figure 1. Geometrical interpretation of multiplying and dividing two numbers

There are several advantages to our approach. Previously, students had errors in line-up of partial sums. In our system, there are as many x's on the right hand side of partial sums as the number of 0's in multiplication digits on the left hand side. For instance partial sum four, 2×1000 has three x's. Therefore chances of leaving any errors is significantly reduced. Secondly, while generating partial sums there was a tendency to stop at any intermediate digit of operand M1.

This idea of adding partial sums is very suitable in base two system because the digits in use are 0 and 1 in base 2 as opposed to decimal system, base 10 in which the digits change from 0 to 9. In base two system, you do not need to multiply digits of M2 with M1 for creating partial sums instead add M1 after appropriate amount of shift. Next we will explain the modifications required in steps to perform division between two numbers that have decimal point fractions. Also, we will illustrate the procedure by giving an example.

Steps to divide D2 divisor into D1 dividend to find quotient and remainder

- Step S(1): Write division expression in-line (this will be clear when you look at the example). Ignore decimal points in both D1 and D2.
- Step S(2): Count number of decimal places on right side after decimal point (DP) in dividend number, p1. If no decimal point then p1 is zero. The place count is not zero only if any digit on rhs of DP is nonzero.
- Step S(3): Count the number of decimal places on right side of decimal point in divisor number, p2. Find difference $p1 - p2$. We will call this as correction for decimal point p. Adjust decimal point in quotient as per correction p from S(2) & S(3).
- Step S(4): Perform division by generating quotient digits as per ordinary division process, multiply divisor by quotient digit and subtract from dividend digits. Continue process till you have a remainder. When you reach a remainder place a decimal point in quotient to find decimal fraction in quotient. (This step will be clear in illustration). You can continue to find more digits of decimal fraction in quotient. You may stop if remainder is zero or wish to stop then.
- Step S(5): Adjust the decimal point position in the quotient. If $p > 0$ (positive) move decimal point to the left . If $p < 0$ (negative) move decimal point to the right after inserting zeroes if needed.

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We will illustrate the process using the same numerical example. Let us divide D1, $562.5884 \div D2, 25.52$
S(5)⇒ Adjust decimal point in quotient by p positions. Answer quotient = 22.045

S(1) $5625884 \div 2552$, **S(2)** count $p1 = 4$, **S(3)** count $p2 = 2$, $p1 - p2 = p = 2$.

S(5)⇒ Adjust decimal point in quotient by $p = 2$.

$$\begin{array}{r}
 \begin{array}{c} p \leftarrow \\ \downarrow \\ 2204\downarrow 5 \end{array} \\
 \hline
 2552 \overline{)5625884} \\
 - 5104xxx \\
 \hline
 05218xx \\
 - x5104xx \\
 \hline
 0011484 \\
 - xx10208 \\
 \hline
 00012760 \\
 - xxx12760 \\
 \hline
 00000000
 \end{array}$$

Here x's are position markers. Notice how position markers wrap around.

From this illustration it is evident that division is reverse process to multiplication. We are subtracting repeated partial products of a quotient digit and the divisor from the dividend. As the steps proceed, we accrued quotient digits. The main advantage of our method is we are certain about position of decimal point in quotient when procedure is finished. Conventionally, a decimal point is placed in quotient by inspection of decimal point position in dividend. Often students, do not write decimal point at correct position in quotient digits and get wrong answer.

In Figure 1 geometrical meaning of multiplying and dividing two numbers is shown. In general product of two numbers represents area of rectangle whose sides are of lengths corresponding to each number. The partial sum produced from multiplying each digit of second number to M1 adds area to the product rectangle. We call this area model of multiplying two numbers because area of product, the rectangle representing result grows as partial sums are generated and added. The division is reverse process than multiplication. The area corresponding to partial products is subtracted from number that we are dividing (the area to begin with) into. The partial product (rectangle) is generated by multiplying a quotient digit with divisor, the number that is dividing the dividend. Thus in division process rectangle is shrinking and in multiplication process, the result rectangle is growing.

References

1. http://www.csci.csusb.edu/schubert/tutorials/csci313/w04/TB_BoothTutorial.pdf
2. <http://people.cs.pitt.edu/~jmisurda/teaching/cs447/examples/Booth%20Example.pdf>

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A Reliable Method to Compute GCF and LCM of Numbers

By Shailesh Kadakia, MSE

Finding the greatest common factors (GCF) and the least common multiple (LCM) for a given set of numbers is a challenge for some students. Here we are presenting a systematic procedure to compute GCF and LCM for provided numbers by using factoring tree. We will illustrate the steps for determining GCF and LCM of two numbers. The steps are as follows.

Step S(1): Draw factor trees for each number by repeated division of the numbers by prime factor numbers.

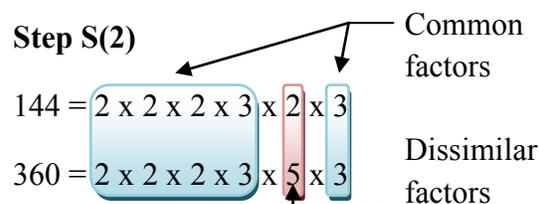
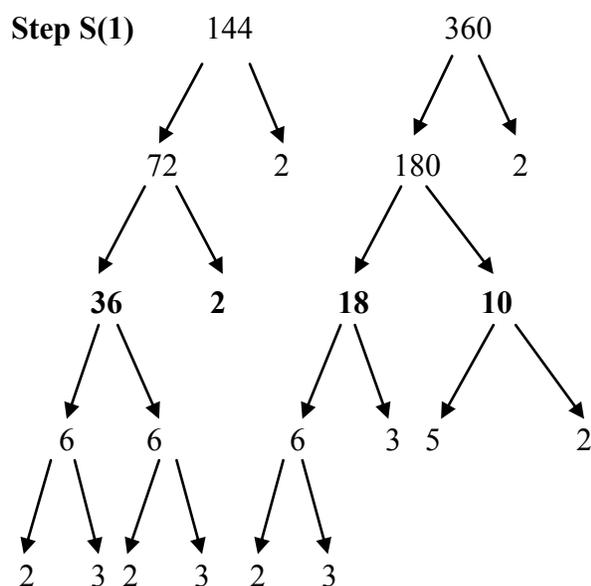
For instance try dividing by 2 for even number. Write quotient below the dividend on one leaf and divisor 2 on another leaf. Try dividing by 3 if digits of number add to number that is divisible by 3, remainder should be zero. Then original number is divisible by 3. Create factor tree using the same process as tree for factor 2. You should continue making tree by using prime factor 5 if number ends with a 5 or a 0. The process of preparing factor tree should be continued till no prime factor can divide the given numbers (This step will be clear in illustration).

Step S(2): Express the numbers as product of all factors by picking leaves from factor tree those do not have a subtree below.

Step S(3): To find GCF, multiply factors that are common factors (CF) of numbers given.

Step S(4): To find LCM, find factors that are not common factors (dissimilar factors DF) in the factors of numbers. Multiply different factors DF and GCF to determine LCM.

Let us illustrate the process for numbers 144 and 360.



Step S(3) Common factors: 2, 2, 2, 3, and 3.

$$\text{GCF} = 2 \times 2 \times 2 \times 3 \times 3 = 72$$

Step S(4) Dissimilar factors (DF): 5 and 2

$$\text{LCM} = 5 \times 2 \times \text{GCF} = 10 \times 72 = 720$$